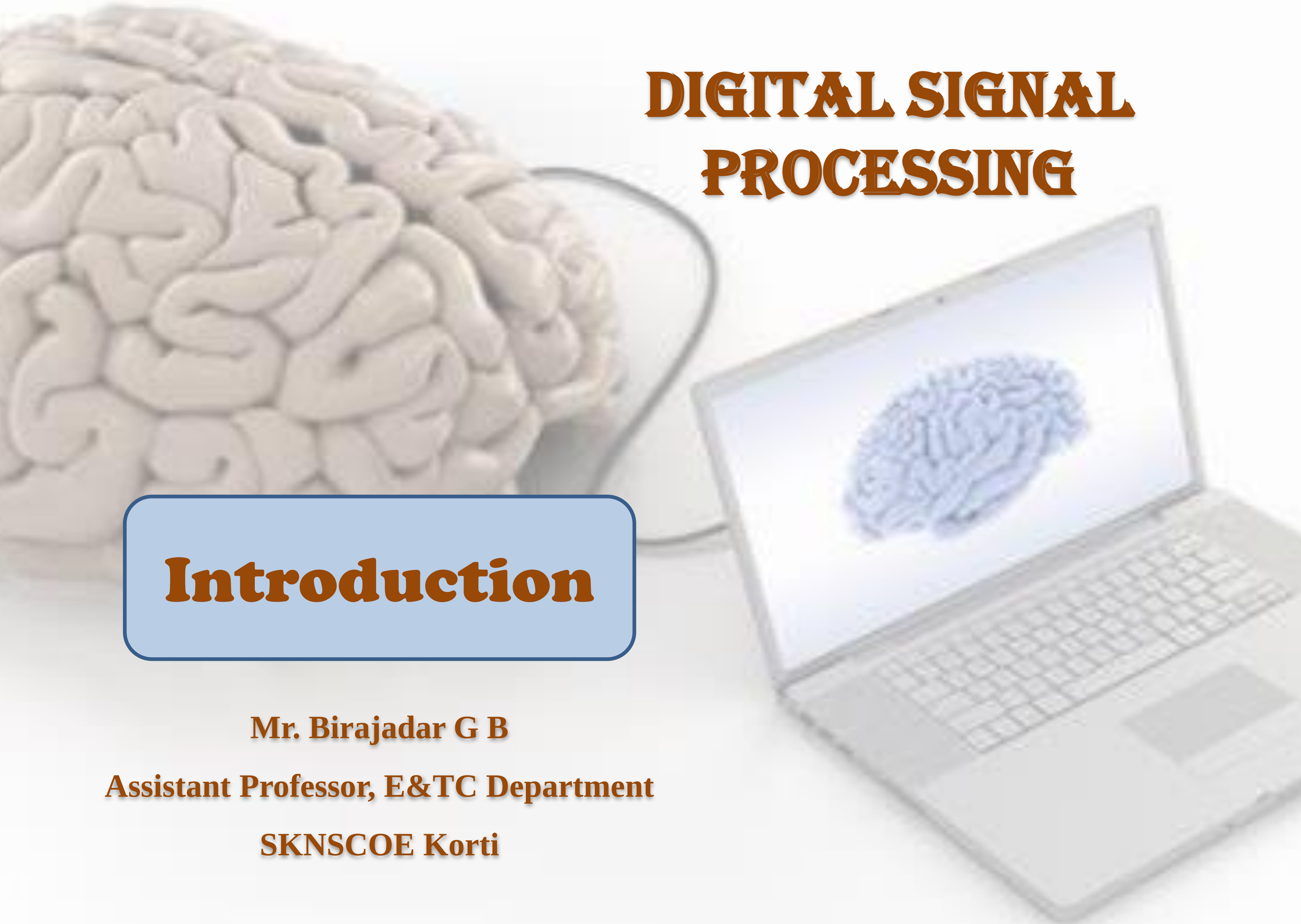

DIGITAL SIGNAL PROCESSING

Introduction

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Course Objectives

Course Name	**: Digital Signal Processing**	**Course Code:**	**: ET313**
Class	**: TY**	**Semester**	**: I**
Academic Year	**: 2022--23**	**Subject Teacher**	**: Birajadar G. B.**

CO No.	Course Outcome Statements	Cognitive Level
ET313.1	**Solve problems based on Correlation and DFT**	**L3: Apply**
ET313.2	Analyze response of the system using linear filtering	L3: Apply
ET313.3	Calculate FFT of the Discrete signal	L3: Apply
ET313.4	Analyze FIR & IIR filter coefficients using different techniques.	L3: Apply
ET313.5	Realize transfer function of FIR & IIR filters using different methods	L3: Apply
ET313.6	Apply concepts of DSP in various applications	L1: Understanding

Digital Signal Processing….. Why

- Analog systems are very susceptible to noise.
- Analog system behavior changes with time because of the change in behavior of analog components like resistors, capacitors, transistors etc.
- Analog systems designed for a purpose can be used only for that purpose and not for any other purpose.
- Digital are less susceptible to noise and if properly designed can remove susceptibility to noise completely.
- Behavior stays same throughout lifetime.
- Flexible because they can be programmed to multiple tasks using a DSP.

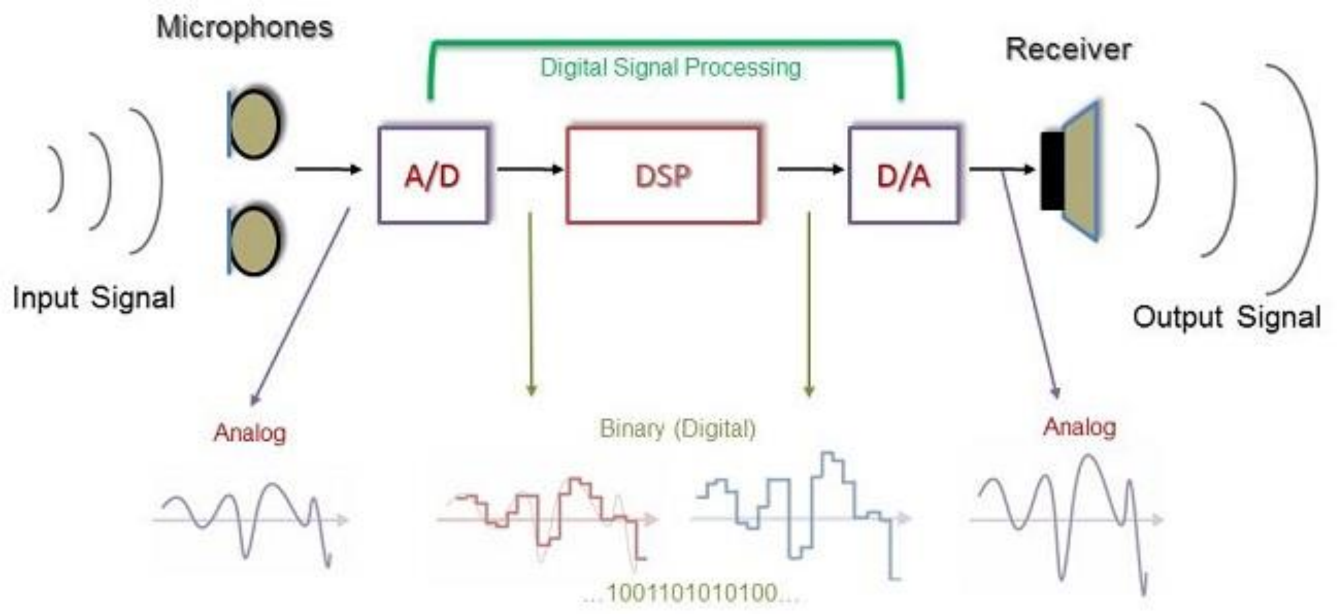
Digital Signal Processing….. Why

- Signals need to be processed so that the information that they contain can be displayed, analyzed, or converted to another type of signal that may be of use.
- In the real-world, analog products detect signals such as sound, light, temperature or pressure and manipulate them.
- Converters such as an A/D converter then take the real-world signal and turn it into the digital format of 1's and 0's.
- DSP takes over by capturing the digitized information and processing it.
- It then feeds the digitized information back for use in the real world, either digitally or in an analog format by going through a D/A converter.
- All of this occurs at very high speeds.

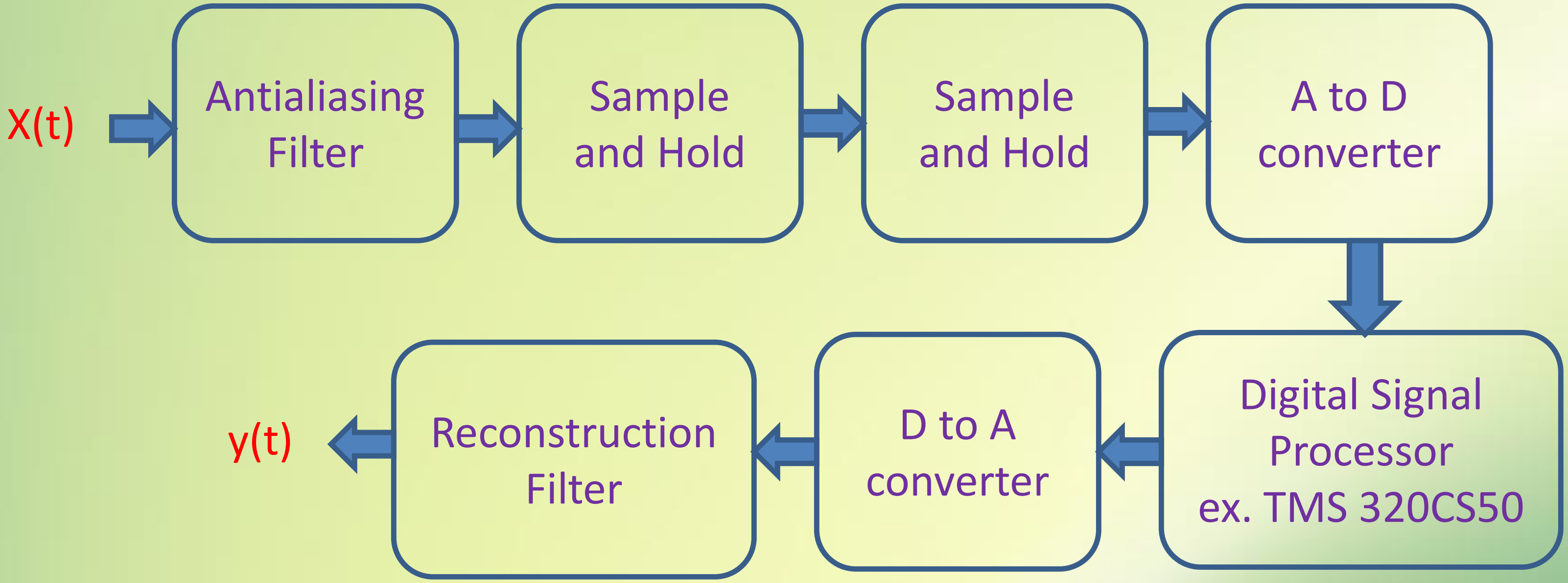
Real Time example MP3 Audio Player

- To illustrate this concept, the diagram below shows how a DSP is used in an MP3 audio player.
- Analog ➡ Digital ➡ DSP ➡ Digital ➡ Analog

MP3 encoding, decoding, volume control, equalization

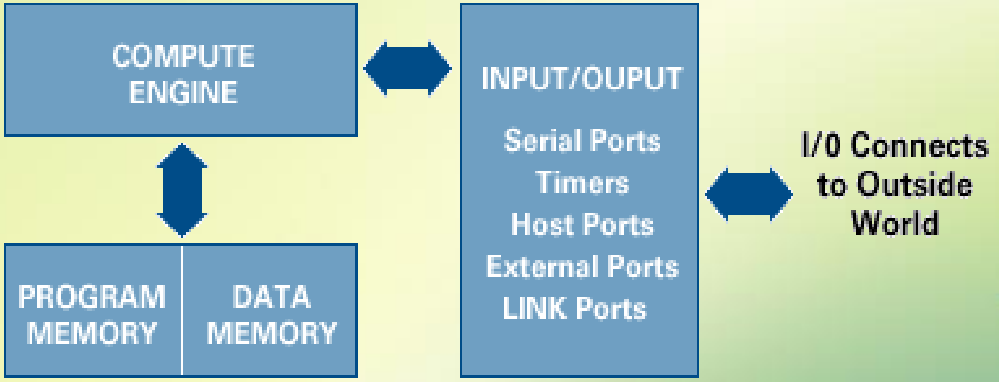


DSP Block Diagram



What is inside DSP ?

- **Program Memory**:
  Stores the programs
- **Data Memory:**
  Stores the information to be processed
- **Compute Engine:**
  Performs the math processing
- **Input/Output:**
  connect to the outside world



Advantages of DSP over Analog SP

- **Flexibility:** Same hardware can be used to do various kind of signal processing operation
- **Repeatability:** The same signal processing operation can be repeated again and again giving same results
- **Accuracy:** Depends on word length, floating or fixed point arithmetic
- **Easy Storage:** Can be easily stored on disk
- **Easy Implementation:** Operations can be easily implemented
- **Cheaper to Implement**

The choice between analog or digital signal processing depends on application. One has to compare design time, size and cost of the implementation.

Disadvantages of DSP over Analog SP

- **System Complexity:**
- **Limited Bandwidth:**
- **More costly hardware:**
- **More power consumption:**

The choice between analog or digital signal processing depends on application. One has to compare design time, size and cost of the implementation.

Applications of DSP

- **Filtering**
- **Speech synthesis** in which white noise (all frequency components present to the same level) is filtered on a selective frequency basis in order to get an audio signal
- **Speech compression and expansion** for use in radio voice communication
- **Speech recognition**
- **Signal analysis**
- **Image processing** like filtering, edge effects, enhancement
- **Modulation** used in telecommunication
- High speed MODEM data communication using **pulse modulation** systems such as FSK, QAM etc
- **Wave form generation**

Applications of DSP

- **Telecommunication** Systems modulation, echo cancellation
- **Consumer electronics** Digital Camera, Digital TV
- **Music** Synthetic instruments, Noise reduction, Audio effects
- **Biomedical** MRI, Ultrasonic imaging, ECG, EEG, MEG
- **Experimental Physics** Sensor data evaluation
- **Aviation** RADAR, radio navigation
- **Image Processing** Image analysis, Pattern recognition
- **Military** Missile Guidance
- **Audio & Speech processing** Speech recognition, Speech Synthesis
- **Instrumentation & Control** Robot control
- **Seismology** Earth quake monitoring, Detection of underground nuclear explosion

Correlation

- **Correlation** is a measure of similarity between two signals.
- There are two types of correlation:

1. Auto correlation

$$\gamma_{xx}(l) = \sum_{-\infty}^{\infty} x(n)x(n-l) \qquad l = 0, \pm 1, \pm 2 \ \ldots\ldots$$

2. Cross correlation

$$\gamma_{xy}(l) = \sum_{-\infty}^{\infty} x(n)y(n-l) \qquad l = 0, \pm 1, \pm 2 \ \ldots\ldots$$

where, Index l represent lag or shift parameter

Properties of Correlation

- For l=0, Autocorrelation defines energy of sequence.

$$\gamma_{xx}(0) = \sum_{-\infty}^{\infty} |x^2(n)|$$

- Correlation possess property of even sequence.

$$\gamma_{xx}(l) = \gamma_{xx}(-l)$$
$$\gamma_{xy}(l) = \gamma_{xy}(-l)$$

- Correlation & Convolution

$$\gamma_{xy}(n) = \mathrm{x}(n) * y(-n)$$

Example

- Given x(n)={1,2,3,4}   h(n) = {1,2,1,2} Find cross correlation.

Steps :

1. Find range of x(n) & h(-n)
2. Calculate range of l
3. Calculate range of n from x(n)
4. For each value of l find correlation values.

$$\gamma_{xy}(l) = \sum_{-\infty}^{\infty} x(n)y(n-l) \qquad l = 0, \pm 1, \pm 2 \ \ldots\ldots$$

Discrete Fourier Transform

- Frequency domain representation of x(n) by samples of its spectrum X(w)
- Discrete Fourier Transform

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \quad k=0,1, \ldots N-1$$

- Inverse Discrete Fourier Transform

$$x(n) = (1/N)\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N} \quad n=0,1, \ldots N-1$$

Properties of DFT

- **Periodicity** — $X(k+N) = X(k)$
- **Linearity** — $DFT[ax_1(n)+bx_2(n)]=aX_1(k)+bX_2(k)$
- **Circular Time Shift** — $DFT[x((n-m))_N]= e^{-j2\pi km/N} X(k)$
- **Time reversal** — $DFT[x((-n))_N]= X(N-k)$
- **Circular Frequency Shift** — $DFT[x(n)e^{j2\pi ln/N}] =X((k-l))_N$
- **Complex Conjugate Property** — $DFT[x^*(n)]=X^*(N-k)$
- **Circular Convolution** — $DFT[x_1(n) \circledN x_2(n)]= X_1(k).X_2(k)$

Multiplication of two DFT's in frequency domain is nothing but circular convolution in time domain.

DFT using Twiddle factor

- Same set of W values that repeat over & over for different values of n

$$W_N^{nk} = e^{-j2\pi kn/N}$$

- Discrete Fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \qquad k=0,1, \ldots N-1$$

- Inverse Discrete Fourier Transform

$$x(n) = (1/N) \sum_{k=0}^{N-1} X(k) W_N^{-nk} \qquad n=0,1, \ldots N-1$$

Properties of Twiddle factor

- **Periodicity**

$$W_N{}^{r} = W_N{}^{r+N}$$

- **Symmetry**

$$W_N{}^{r} = -W_N{}^{r\pm(N/2)}$$

Circular Convolution

- **Graphical Method / Concentric Circle Method**

  - ✓ **Given two sequences x(n) & h(n).**
  - ✓ **Graph x(n) around outer circle in counter clockwise direction.**
  - ✓ **Graph h(n) around inner circle in clockwise direction.**
  - ✓ **Multiply corresponding samples on two circles.**
  - ✓ **Rotate inner circle one sample at a time in counter clockwise direction.**
  - ✓ **Repeat step no 4**

Circular Convolution

- **Matrix Multiplication Method**
  - ✓ **One of the given sequences is repeated via circular shift of one sample at a time to form a N X N matrix.**
  - ✓ **The other sequence is represented as column matrix.**
  - ✓ **The multiplication of two matrices give the result of circular convolution.**

- $$\begin{bmatrix} x(0) & x(3) & x(2) & x(1) \\ x(1) & x(0) & x(3) & x(2) \\ x(2) & x(1) & x(0) & x(3) \\ x(3) & x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

Circular Convolution

- **Tabular Method**
  - ✓ **One of the given sequences is rotated circularly.** **g(n)**
  - ✓ **The other sequence is rotated linearly.** **h(n)**
  - ✓ **Addition of each column gives output .**

$$
\begin{array}{c}
\\ + \\ + \\ +
\end{array}
\begin{bmatrix}
g(0)h(0) & g(1)h(0) & g(2)h(0) & g(3)h(0) \\
g(3)h(1) & g(0)h(1) & g(1)h(1) & g(2)h(1) \\
g(2)h(2) & g(3)h(2) & g(0)h(2) & g(1)h(2) \\
g(1)h(3) & g(2)h(3) & g(3)h(3) & g(0)h(3)
\end{bmatrix}
$$

$$y(0) \quad y(1) \quad y(2) \quad y(3)$$

Circular Convolution

- **DFT IDFT Method**
  - ✓ **Find DFT of both sequences. X(k) , H(k)**
  - ✓ **Multiply both DFTs. Y(k)= X(k).H(k).**
  - ✓ **Find inverse DFT of multiplication answer. Y(n) = IDFT (Y(k))**

Linear Vs Circular Convolution

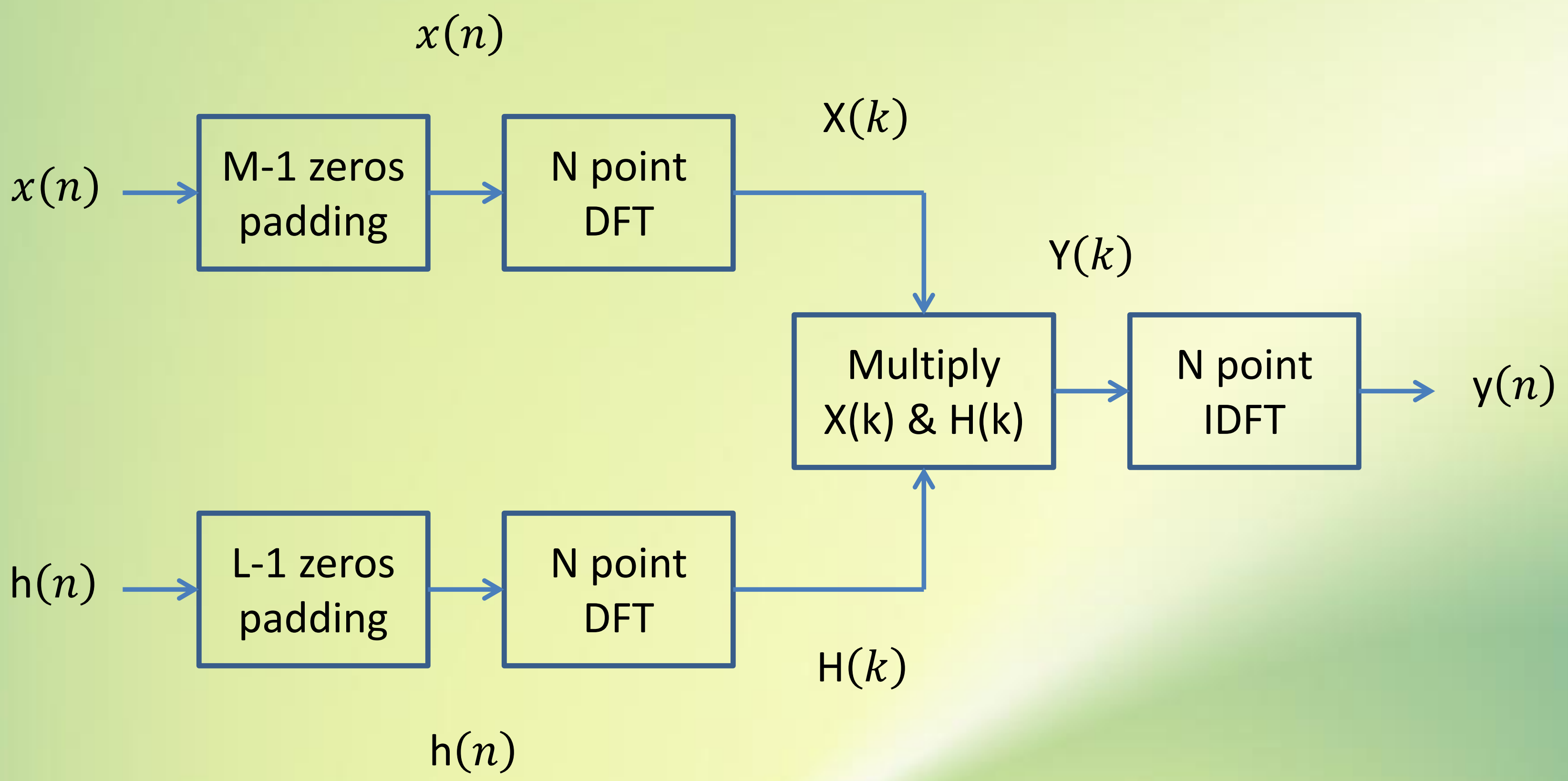
❖ Linear Convolution

1. If x(n) is sequence of L no of samples & h(n) is sequence of M no of samples, output y(n) will contain N= L+M-1 samples.
2. $y(n) = x(n)*h(n)$
3. Used to find response of linear filter.
4. Zero padding is not necessary.

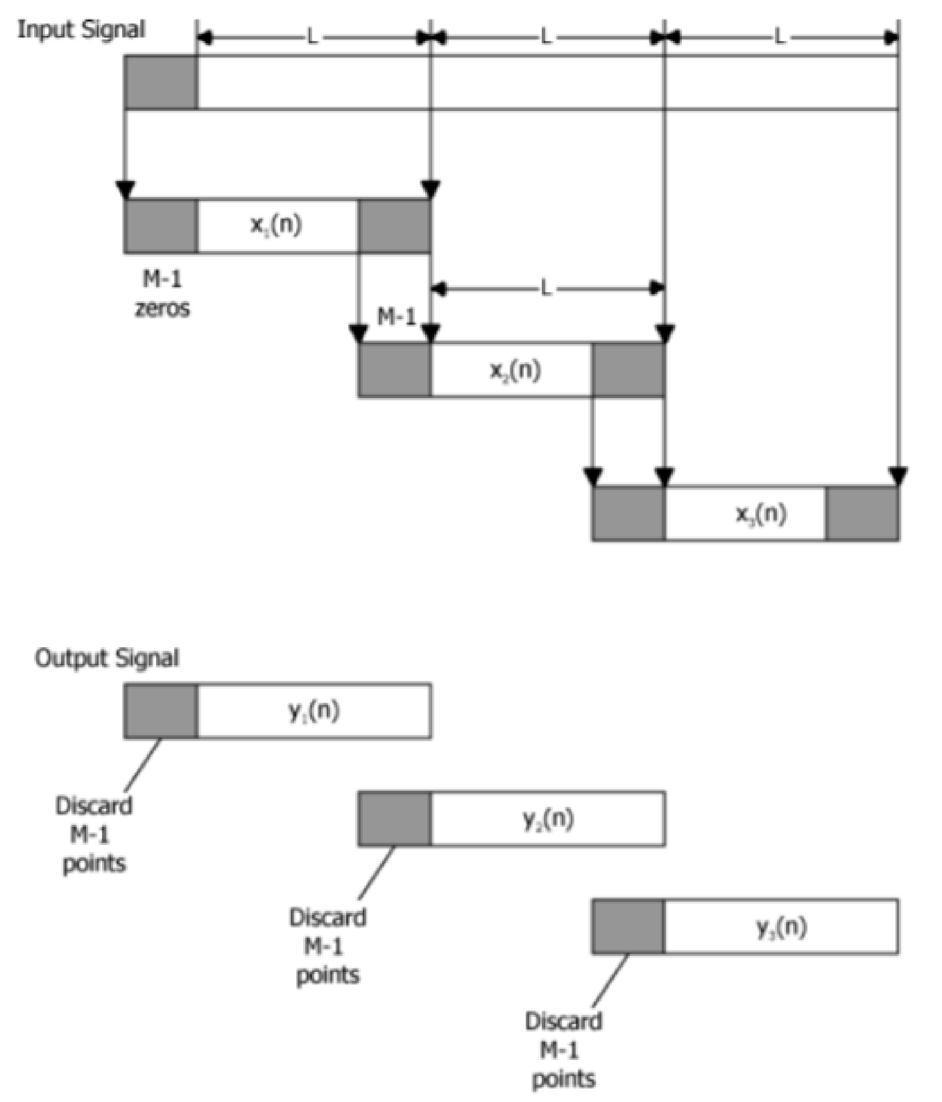
❖ Circular Convolution

1. If x(n) is sequence of L no of samples & h(n) is sequence of M no of samples, output y(n) will contain N= max(L,M) samples.
2. $y(n) = x(n) \textcircled{N} h(n)$
3. Can't be used to find response of linear filter.
4. Zero padding is necessary.

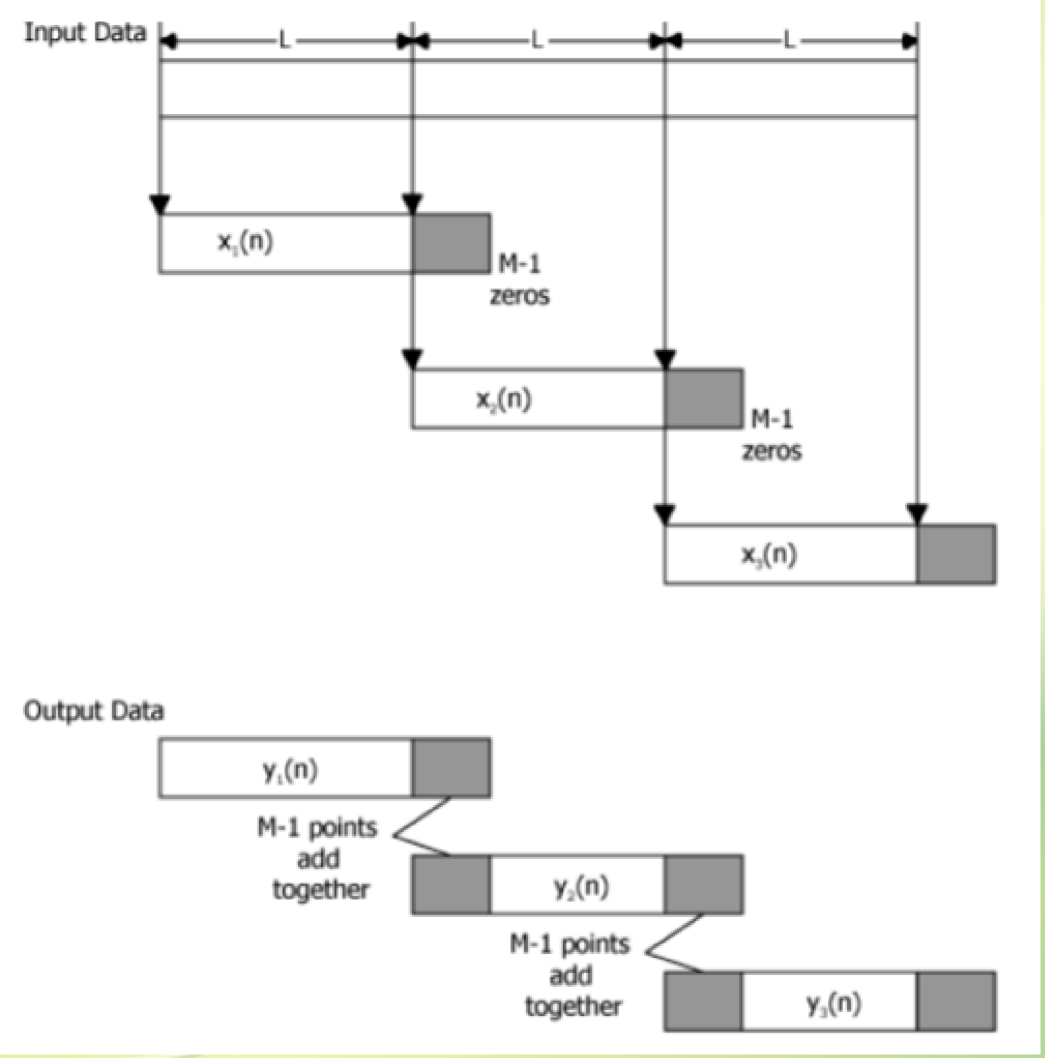
Linear Convolution using DFT IDFT



Overlap Save Method



Overlap ADD Method



Overlap Save Vs Overlap Add

❖ Overlap Save

1. Size of input data block L+M-1
2. Each data block consists of last M-1 elements of previous data block, followed by new L data points
3. In each output block, first M-1 elements are corrupted due to aliasing.
4. To get output, first M-1 data points are discarded from each output block.

❖ Overlap Add

1. Size of input data block L
2. Each data block consists of L points. We append M-1 zeros to each block.
3. There is no aliasing.
4. To get output, last M-1 points from each block are added to first M-1 points of successive output block.

Direct evaluation of DFT

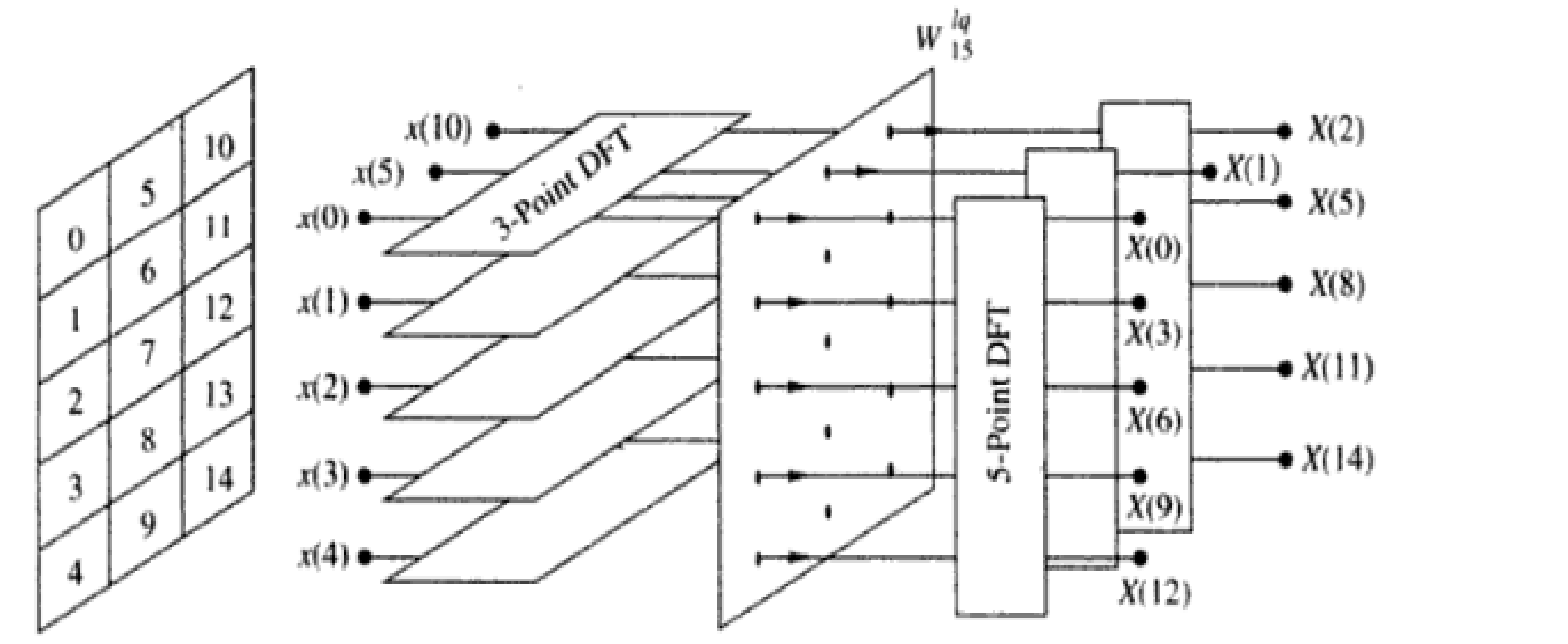
- **To evaluate one value of X(k)**
  - ❑ No. of complex multiplications = N
  - ❑ No. of complex additions = N -1
  - ❑ No. of real multiplications = 4 N
  - ❑ No. of real additions = 4 N - 2

- **To evaluate N point DFT X(k)**
  - ❑ No. of complex multiplications = N^2
  - ❑ No. of complex additions = N(N -1)
  - ❑ No. of real multiplications = 4 N^2
  - ❑ No. of real additions = N (4 N – 2)

Divide & Conquer Approach

Consider sequence of length N = 15

N = L * M = 5 * 3



Divide & Conquer Approach

- **Algorithm 1**

  - Store the signal column wise.
  - Compute M point DFT of each row.
  - Multiply resulting array by phase factor W^{lq}_N
  - Compute L point DFT of each column.
  - Read resulting array row wise

- **Algorithm 2**

  - Store the signal row wise.
  - Compute L point DFT of each column.
  - Multiply resulting array by phase factor W^{pm}_N
  - Compute M point DFT of each row.
  - Read resulting array column wise

Bit reversal algorithm

I/p sample index	Binary representation	Bit reversed Binary	Bit reversed I/P index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

Decimation in Time FFT algorithm

- No. of input samples $N= 2^M$ where M is integer.
- Input is bit reversed & output is in natural order.
- No. of stages in flow graph = $M = \log_2 N$.
- No. of butterflies in each stage = $N / 2$.
- Input Output samples are separated by $2^{(m-1)}$ samples where m is stage index.
- No. of complex multiplications = $(N /2) \log_2 N$.
- No. of complex additions = $N \log_2 N$.
- Twiddle factor exponents are function of stage index m

  $k= Nt/ 2^m \qquad t= 0, 1, \ldots\ldots, 2^{m-1} -1$
- No of sections of butterflies in each stage = 2^{M-m}
- Exponent repeat factor (ERF) = 2^{M-m}

Decimation in Frequency FFT algorithm

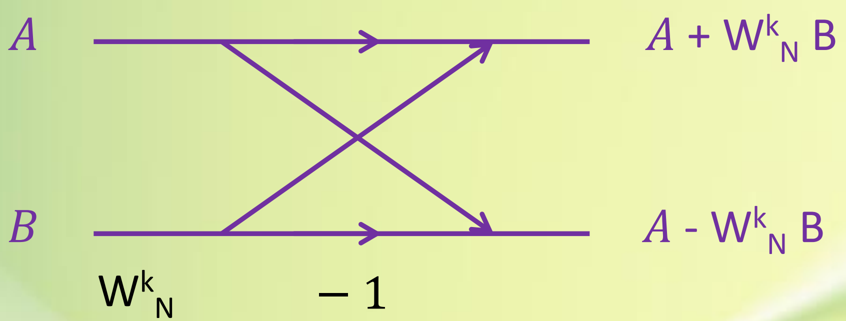
- No. of input samples $N = 2^M$ where M is integer.
- Input is in natural order & output is bit reversed.
- No. of stages in flow graph = $M = \log_2 N$.
- No. of butterflies in each stage = $N / 2$.
- Input Output samples are separated by $2^{(M-m)}$ samples where m is stage index.
- No. of complex multiplications = $(N/2) \log_2 N$.
- No. of complex additions = $N \log_2 N$.
- Twiddle factor exponents are function of stage index m

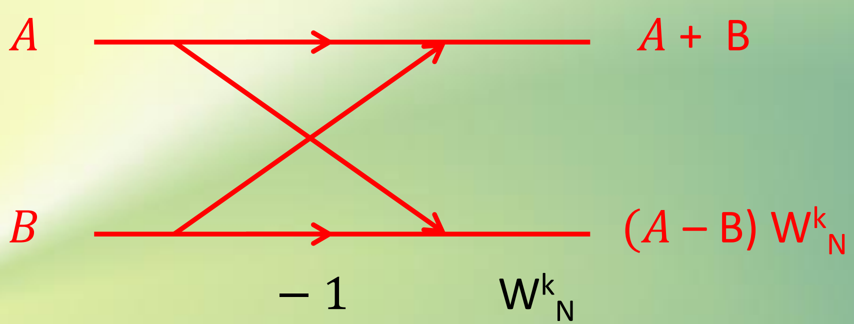
  $k = Nt / 2^{M-m+1}$ $\quad t = 0, 1, \ldots\ldots, 2^{M-m} - 1$
- No of sections of butterflies in each stage = 2^{m-1}
- Exponent repeat factor (ERF) = 2^{m-1}

Similarities & Differences in DIT & DIF

- ✓ Same no of computations are required.
- ✓ Both can be done in place.
- ✓ Both need to perform bit reversal at some place.

- ✓ In DIT Input is bit reversed & output is in natural order.
- ✓ In DIF Input is in natural order & output is bit reversed.
- ✓ Basic operation





IDFT using FFT algorithm

- Take complex conjugate of given sequence.
- Depend on DIT or DIF arrange input sequence.
- Apply respective algorithm.
- Depend on DIT or DIF read output sequence.
- Output calculated is in terms of N . $\boldsymbol{x}^{*}(\boldsymbol{n})$
- To get final output sequence, divide output by N & take complex conjugate.


THANK YOU :: ::